

Risoluzione Esercizi Deduzione Naturale [AU]

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Esercizio 1: $(A \wedge B) \vee A \vdash A$.

$$\frac{\frac{[A \wedge B]^1}{A} \wedge E \quad [A]^2 \quad (A \wedge B) \vee A}{A} \vee E(1)(2)$$

Esercizio 2: $A \vdash (A \wedge B) \vee A$.

$$\frac{A}{(A \wedge B) \vee A} \vee I$$

Esercizio 3: $A \rightarrow B, B \rightarrow A \vdash (A \vee B) \rightarrow (A \wedge B)$.

$$\frac{\frac{[A]^1 \quad A \rightarrow B}{B} \rightarrow E \quad [A]^1 \quad \wedge I \quad \frac{[B]^2 \quad B \rightarrow A}{A} \rightarrow E \quad [B]^2 \quad \wedge I}{\frac{A \wedge B}{(A \vee B) \rightarrow (A \wedge B)} \rightarrow I(3)} \vee E(1)(2)$$

Esercizio 4: $\neg A \vee B \vdash A \rightarrow B$

$$\frac{\frac{[\neg A]^1 \quad [A]^3}{\perp} \neg E \quad \frac{\perp}{A \rightarrow B} \rightarrow I(3) \quad \frac{[B]^2}{A \rightarrow B} \rightarrow I}{A \rightarrow B} \vee E(1)(2)$$

Esercizio 5: $A \vee B, B \vee C \vdash (A \wedge B) \vee C$

$$\frac{\frac{[A]^1 \quad [B]^2}{A \wedge B} \wedge I \quad \frac{[C]^3}{(A \wedge B) \vee C} \vee I}{\frac{(A \wedge B) \vee C}{(A \wedge B) \vee C} \vee E(2)(3)} \vee I \quad \frac{[C]^3}{(A \wedge B) \vee C} \vee I \quad A \vee C}{(A \wedge B) \vee C} \vee E(1)(3)$$

Esercizio 7: $A \vee B \vdash B \vee A$

$$\frac{\frac{[A]^1}{B \vee A} \vee I \quad \frac{[B]^2}{B \vee A} \vee I}{B \vee A} \vee E(1)(2)$$

Esercizio 8: $A \wedge B \vdash B \wedge A$

$$\frac{\frac{A \wedge B}{B} \wedge E \quad \frac{A \wedge B}{A} \wedge E}{B \wedge A} \wedge I$$

Esercizio 9: $A \wedge (B \vee C) \vdash (A \wedge B) \vee (A \wedge C)$

$$\frac{\frac{\frac{A \wedge (B \vee C)}{A} \wedge E \quad [B]^1}{A \wedge B} \wedge I \quad \frac{A \wedge (B \vee C)}{A} \wedge E \quad [C]^2}{A \wedge C} \wedge I}{\frac{(A \wedge B) \vee (A \wedge C)}{(A \wedge B) \vee (A \wedge C)} \vee I} \vee I \quad B \vee C}{(A \wedge B) \vee (A \wedge C)} \vee E(1)(2)$$

Esercizio 10: $\vdash A \rightarrow \neg\neg A$

$$\frac{\frac{\frac{[A]^1 \quad [\neg A]^2}{\perp} \neg E}{\neg\neg A} \neg I(2)}{A \rightarrow \neg\neg A} \rightarrow I(1)}$$

Esercizio 11: $\vdash (A \rightarrow (B \rightarrow C)) \rightarrow ((A \wedge B) \rightarrow C)$

$$\frac{\frac{\frac{\frac{[A \wedge B]^1}{A} \wedge E}{B \wedge C} \wedge E}{A \rightarrow (B \rightarrow C)} \rightarrow E \quad \frac{[A \wedge B]^1}{B} \wedge E}{\frac{C}{(A \wedge B) \rightarrow C} \rightarrow I(1)} \rightarrow I$$

Esercizio 12: $\vdash ((A \wedge B) \rightarrow C) \rightarrow (A \rightarrow (B \rightarrow C))$

$$\frac{\frac{\frac{[A]^2 \quad [B]^1}{A \wedge B} \wedge I}{[(A \wedge B) \rightarrow C]^3} \rightarrow E}{\frac{\frac{C}{B \rightarrow C} \rightarrow I(1)}{A \rightarrow (B \rightarrow C)} \rightarrow I(2)} \rightarrow I(3)$$

Esercizio 13: $\vdash \neg A \rightarrow \neg\neg\neg A$

$$\frac{\frac{\frac{[\neg A]^1 \quad [\neg\neg A]^2}{\perp} \neg E}{\neg\neg\neg A} \neg I(2)}{\neg A \rightarrow \neg\neg\neg A} \rightarrow I(1)}$$

Esercizio 14: $(A \vee B) \rightarrow (A \wedge B) \vdash A \rightarrow B$

$$\frac{\frac{[A]^1}{A \vee B} \vee I}{\frac{\frac{A \wedge B}{B} \wedge E}{A \rightarrow B} \rightarrow I(1)} \rightarrow E$$

Esercizio 15: $\vdash A \rightarrow (B \rightarrow A)$

$$\frac{\frac{[A]^1}{B \rightarrow A} \rightarrow I}{A \rightarrow (B \rightarrow A)} \rightarrow I(1)$$

Esercizio 16: $A \rightarrow \neg A \vdash \neg A$

$$\frac{\frac{[A]^1 \quad A \rightarrow \neg A}{\neg A} \rightarrow E}{\frac{[A]^1}{\perp} \neg E} \neg I(1)$$

Esercizio 17: $\neg A \vee \neg B \vdash A \rightarrow B$

$$\frac{\frac{\frac{\neg[A]^1 \quad [A]^2}{\perp} \neg E}{\frac{B}{A \rightarrow B} \rightarrow I(2)} \rightarrow I(1) \quad \frac{[B]^3}{A \rightarrow B} \rightarrow I}{\frac{\neg A \vee A}{A \rightarrow B} \vee E(1)(3)}$$

Esercizio 18: $(\neg A \vee \neg B) \vdash \neg(A \wedge B)$

$$\frac{\frac{\frac{[A \wedge B]^1}{A} \wedge E}{\frac{\perp}{\neg(A \wedge B)} \neg I(1)} \neg E \quad \frac{\frac{[A \wedge B]^1}{B} \wedge E}{\frac{\perp}{\neg(A \wedge B)} \neg I(1)} \neg E}{\frac{\neg A \vee \neg B}{\neg(A \wedge B)} \vee E(2)(3)}$$

Esercizio 19: $A \rightarrow B \vdash \neg(A \wedge \neg B)$

$$\frac{\frac{\frac{[A \wedge \neg B]^1}{A} \wedge E \quad A \rightarrow B \rightarrow E \quad [A \wedge \neg B]^1 \wedge E}{B \quad \neg B} \neg E}{\perp} \neg I(1)$$

Esercizio 20: $A \wedge B \vdash \neg(\neg A \vee \neg B)$

$$\frac{\frac{\frac{[A \wedge B]^1}{A} \wedge E \quad [\neg A]^1 \neg E}{\perp} \neg E \quad \frac{\frac{[A \wedge B]^1}{B} \wedge E \quad [\neg B]^2 \neg E}{\perp} \neg E}{[\neg A \vee \neg B]^3 \vee E(1)(2)} \neg I(3)$$

Esercizio 21: $\neg(A \rightarrow B) \vdash A$

$$\frac{\frac{\frac{[A]^1 \quad [\neg A]^2}{A \rightarrow B} \rightarrow I(1) \quad \perp \quad \perp}{\neg(A \rightarrow B)} \neg E}{\perp} \text{RAA}(2)$$

Esercizio 22: $((A \rightarrow B) \rightarrow A) \vdash A$

$$\frac{\frac{\frac{[A \rightarrow B]^1 \quad (A \rightarrow B) \rightarrow A \rightarrow E}{A} \rightarrow E \quad [\neg A]^2 \neg E}{\perp} \neg I(1) \quad \frac{\frac{[\neg A]^2 \quad [A]^3}{A \rightarrow B} \rightarrow I(3) \quad \perp \quad \perp}{\neg(A \rightarrow B)} \neg E}{\perp} \text{RAA}(2)$$

Esercizio 23: $\neg(A \wedge B) \vdash \neg A \vee \neg B$

$$\frac{\frac{\frac{[\neg A]^1}{\neg A \vee \neg B} \vee I \quad [\neg(\neg A \vee \neg B)]^3 \neg E}{\perp} \neg I(1) \quad \frac{\frac{[\neg B]^2}{\neg A \vee \neg B} \vee I \quad [\neg(\neg A \vee \neg B)]^3 \neg E}{\perp} \neg I(2)}{\frac{\perp}{\neg A} \neg E \quad \frac{\perp}{\neg B} \neg E} \text{RAA}(1) \quad \text{RAA}(2)$$

$$\frac{\frac{\perp}{A} \text{RAA}(1) \quad \frac{\perp}{B} \text{RAA}(2)}{A \wedge B} \wedge I \quad \frac{\perp}{\neg(A \wedge B)} \neg E$$

Esercizio 24: $\neg(A \wedge \neg B) \vdash A \rightarrow B$

$$\frac{\frac{\frac{[\neg A]^1}{\neg A \vee B} \vee I \quad [\neg(\neg A \vee B)]^3 \neg E}{\perp} \neg I(1) \quad \frac{\frac{[B]^2}{\neg A \vee B} \vee I \quad [\neg(\neg A \vee B)]^3 \neg E}{\perp} \neg I(2)}{\frac{\perp}{\neg A} \neg E \quad \frac{\perp}{\neg B} \neg E} \text{RAA}(1) \quad \text{RAA}(2)$$

$$\frac{\frac{[\neg A]^5 \quad [A]^4}{A \rightarrow B} \rightarrow I(4) \quad \frac{[B]^6}{A \rightarrow B} \rightarrow I}{\perp} \neg E \quad \frac{\perp}{\neg(A \wedge \neg B)} \neg E$$

$$\frac{\perp}{A \rightarrow B} \text{RAA}(3) \quad \vee E(5)(6)$$

Esercizio 25: $\neg(\neg A \vee \neg B) \vdash A \wedge B$

$$\frac{\frac{\frac{[\neg A]^1}{\neg A \vee \neg B} \vee I \quad \neg(\neg A \vee \neg B)}{\perp} \text{RAA(1)} \quad \frac{\frac{[\neg B]^2}{\neg A \vee \neg B} \vee I \quad \neg(\neg A \vee \neg B)}{\perp} \text{RAA(2)}}{A \wedge B} \wedge I$$

Esercizio 26: $\vdash (\neg A \rightarrow A) \rightarrow A$

$$\frac{\frac{\frac{[A]^1}{(\neg A \rightarrow A) \rightarrow A} \rightarrow I \quad [\neg((\neg A \rightarrow A) \rightarrow A)]^3}{\perp} \neg I(1) \quad \frac{[\neg A \rightarrow A]^2}{(\neg A \rightarrow A) \rightarrow A} \rightarrow E}{\frac{A}{(\neg A \rightarrow A) \rightarrow A} \rightarrow I(2) \quad [\neg((\neg A \rightarrow A) \rightarrow A)]^3}{\perp} \text{RAA(3)}}{(\neg A \rightarrow A) \rightarrow A} \rightarrow E$$

Esercizio 27: $\neg A \rightarrow \neg B \vdash B \rightarrow A$

$$\frac{\frac{\frac{\neg A \rightarrow \neg B}{\neg B} \quad [\neg A]^1}{\perp} \rightarrow E \quad [B]^3}{\perp} \neg I(1) \quad \frac{[\neg A]^2}{B \rightarrow A} \rightarrow I(3)}{B \rightarrow A} \rightarrow E$$

Esercizio 28: $\neg A \rightarrow B \vdash A \vee B$

$$\frac{\frac{\frac{[\neg A]^4}{\neg A \vee B} \rightarrow E \quad \frac{B}{A \vee B} \vee I}{\perp} \neg I(2) \quad \frac{[A]^5}{A \vee B} \vee I \quad \frac{\frac{[\neg A]^2}{\neg A \vee A} \vee I \quad [\neg(\neg A \vee A)]^1}{\perp} \text{RAA(3)} \quad \frac{[\neg A]^3}{A \vee \neg A} \vee I \quad [\neg(\neg A \vee A)]^1}{\perp} \text{RAA(1)} \quad \vee E(4)(5)}{A \vee B} \rightarrow E$$

Esercizio 29: $\neg(\neg A \wedge \neg B) \vdash A \vee B$

$$\frac{\frac{\frac{[A]^1}{A \vee B} \vee I \quad [\neg(A \vee B)]^3}{\perp} \neg I(1) \quad \frac{\frac{[B]^2}{A \vee B} \vee I \quad [\neg(A \vee B)]^3}{\perp} \neg I(2)}{\neg A \wedge \neg B} \wedge I \quad \frac{\perp}{\neg(\neg A \wedge \neg B)} \neg E}{\frac{\perp}{A \vee B} \text{RAA(3)}} \rightarrow E$$

Esercizio 30: $\neg A \rightarrow B, B \rightarrow C, \neg C \vdash A$

$$\frac{\frac{[\neg A]^1}{B} \rightarrow E \quad \frac{\neg A \rightarrow B}{B \rightarrow C} \rightarrow E}{C} \rightarrow E \quad \frac{\perp}{A} \text{RAA(1)} \quad \neg C \neg E$$

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