CS 267: Automated Verification

Lecture 2: Linear vs. Branching time. Temporal Logics: CTL, CTL*. CTL model checking algorithm. Counter-example generation.

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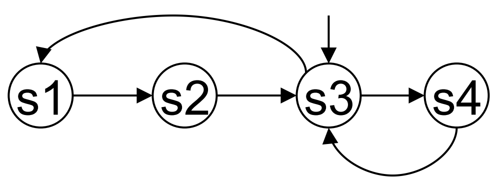
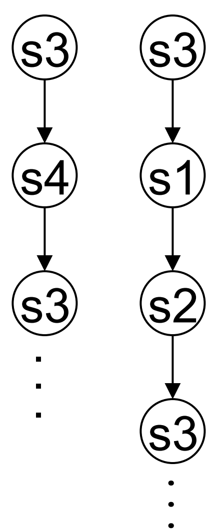
Linear Time vs. Branching Time

- In linear time logics we look at the execution paths individually
- In branching time logics we view the computation as a tree
  - computation tree: unroll the transition relation

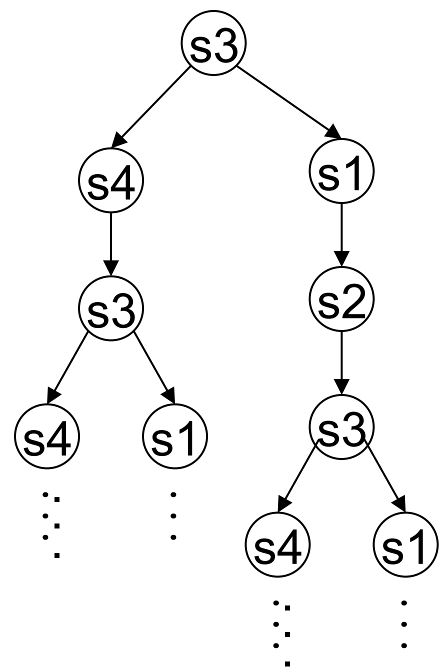
Transition System



Execution Paths



Computation Tree



Computation Tree Logic (CTL)

- In CTL we quantify over the paths in the computation tree
- We use the same four temporal operators: X, G, F, U
- However we attach path quantifiers to these temporal operators:
  - A : for all paths
  - E : there exists a path
- We end up with eight temporal operators:
  - AX, EX, AG, EG, AF, EF, AU, EU

CTL Semantics

Given a state s and CTL properties p and q

$s \models p$	iff	$L(s, p) = \text{True}$, where $p \in AP$
$s \models \neg p$	iff	not $s \models p$
$s \models p \wedge q$	iff	$s \models p$ and $s \models q$
$s \models p \vee q$	iff	$s \models p$ or $s \models q$

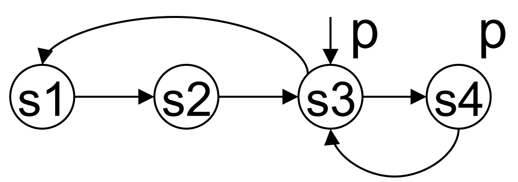
$s_0 \models EX\ p$	iff	there exists a path $s_0, s_1, s_2, \ldots$ such that $s_1 \models p$
$s_0 \models AX\ p$	iff	for all paths $s_0, s_1, s_2, \ldots, s_1 \models p$

CTL Semantics

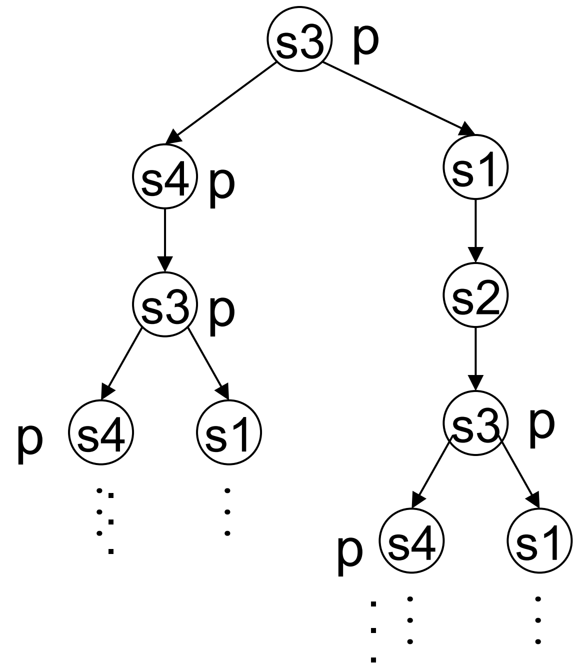
s_0 |= EG p	iff	there exists a path s_0, s_1, s_2, ... such that for all $i \geq 0$, s_i |= p
s_0 |= AG p	iff	for all paths s_0, s_1, s_2, ..., for all $i \geq 0$, s_i |= p
s_0 |= EF p	iff	there exists a path s_0, s_1, s_2, … such that there exists an $i \geq 0$ such that s_i |= p
s_0 |= AF p	iff	for all paths s_0, s_1, s_2, ..., there exists an $i \geq 0$, such that, s_i |= p
s_0 |= p EU q	iff	there exists a path s_0, s_1, s_2, ..., such that, there exists an $i \geq 0$ such that s_i |= q and for all $0 \leq j < i$, s_j |= p
s_0 |= p AU q	iff	for all paths s_0, s_1, s_2, ..., there exists an $i \geq 0$ such that s_i|=q and for all $0 \leq j < i$, s_j|=p

CTL Properties

Transition System



Computation Tree



s3 |= p
s4 |= p
s1 |= ¬ p
s2 |= ¬ p

s3 |= EX p
s3 |= EX ¬ p
s3 |= ¬ AX p
s3 |= ¬ AX ¬ p
s3 |= EG p
s3 |= ¬ EG ¬ p
s3 |= AF p
s3 |= EF ¬ p
s3 |= ¬ AF ¬ p

CTL Equivalences

- CTL basis: EX, EU, EG

$AX\ p = \neg\ EX\ \neg p$

$AG\ p = \neg\ EF\ \neg p$

$AF\ p = \neg\ EG\ \neg p$

$p\ AU\ q = \neg((\neg q\ EU\ (\neg p \wedge \neg q)) \vee EG\ \neg\ q)$

$EF\ p = True\ EU\ p$

- Another CTL basis: EX, EU, AU

CTL Model Checking

- Given a transition system T= (S, I, R) and a CTL property p
  T |= p   iff   for all initial state $s \in I$, s |= p

Model checking problem: Given a transition system T and a CTL property p, determine if T is a model for p (i.e., if T |=p)

For example:

T |=? AG ($\neg$ (pc1=c $\wedge$ pc2=c))

T |=? AG(pc1=w $\Rightarrow$ AF(pc1=c)) $\wedge$ AG(pc2=w $\Rightarrow$ AF(pc2=c))

- Question: Are CTL and LTL equivalent?

CTL vs. LTL

- CTL and LTL are not equivalent

  – There are properties that can be expressed in LTL but cannot be expressed in CTL
    - For example: FG p

  – There are properties that can be expressed in CTL but cannot be expressed in LTL
    - For example: AG(EF p)

- Hence, expressive power of CTL and LTL are not comparable

CTL*

- CTL* is a temporal logic which is strictly more powerful than CTL and LTL

- CTL* also uses the temporal operators X, F, G, U and the path quantifiers A and E, but temporal operators can also be used without path quantifiers

CTL*

- CTL and CTL* correspondence
  - Since and CTL property is also a CTL* property, CTL* is clearly as expressive as CTL
- Any LTL f property corresponds to the CTL* property A f
  - i.e., LTL properties have an implicit “for all paths” quantifier in front of them
  - Note that, according to our definition, an LTL property f holds for a transition system T, if and only if, for all execution paths of T, f holds
  - So, LTL property f holds for the transition system T if and only if the CTL* property A f holds for all initial states of T

CTL*

- CTL* is more expressive than CTL and LTL
- Following CTL* property cannot be expressed in CTL or LTL
  - $A(FG\ p) \vee AG(EF\ p)$

Model Checking Algorithm for Finite State Systems

[Clarke and Emerson 81], [Queille and Sifakis 82]

CTL Model checking problem: Given a transition system T = (S, I, R), and a CTL formula f, does the transition system satisfy the property?

CTL model checking problem can be solved in

$O(|f| \times (|S|+|R|))$

Note that the complexity is linear in the size of the formula and the transition system

– Recall that the size of the transition system is exponential in the number of variables and concurrent components (this is called the ***state space explosion*** problem)

CTL Model Checking Algorithm

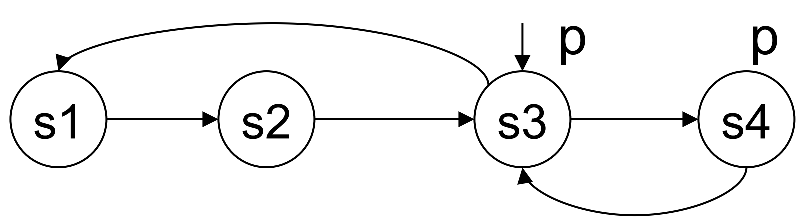
- Translate the formula to a formula which uses the basis
  - EX p, EG p, p EU q
- Start from the innermost subformulas
  - ***Label the states in the transition system with the subformulas that hold in that state***
    - Initially states are labeled with atomic properties
- Each (temporal or boolean) operator has to be processed once
- Processing of each operator takes O(|S|+|R|)

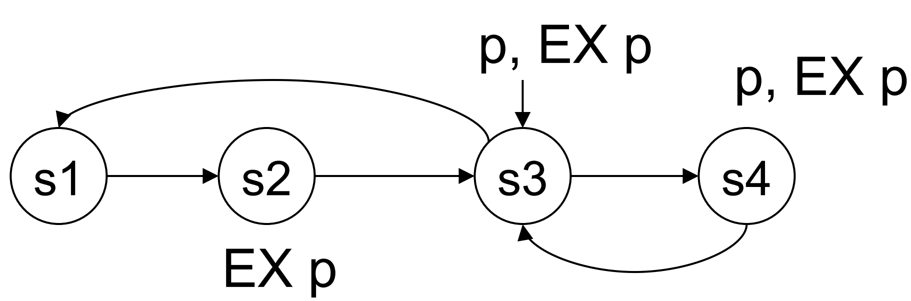
CTL Model Checking Algorithm

- Boolean operators are easy
  - $\neg p$: Each state which is not labeled with p should be labeled with $\neg p$
  - $p \wedge q$: Each state which is labeled with both p and q should be labeled with $p \wedge q$
  - $p \vee q$: Each state which is labeled with p or q should be labeled with $p \vee q$

CTL Model Checking Algorithm: EX p

- EX p is easy to do in O(|S|+|R|)
  - All the nodes which have a next state labeled with p should be labeled with EX p

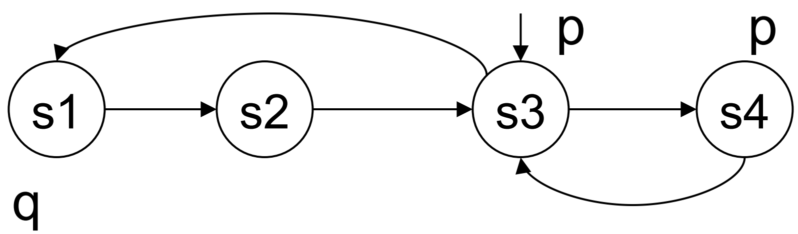


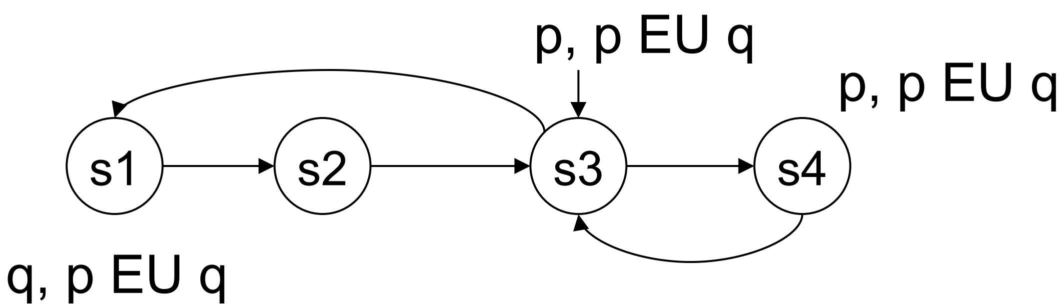


CTL Model Checking Algorithm: p EU q

- p EU q: Find the states which are the source of a path where p U q holds
  - Find the nodes that reach a node that is labeled with q by a path where each node is labeled with p
    - Label such nodes with p EU q
  - It is a reachability problem which can be solved in O(|S|+|R|)
    - First label the nodes which satisfy q with p EU q
    - For each node labeled with p EU q, label all its previous states that are labeled with p with p EU q

CTL Model Checking Algorithm: p EU q

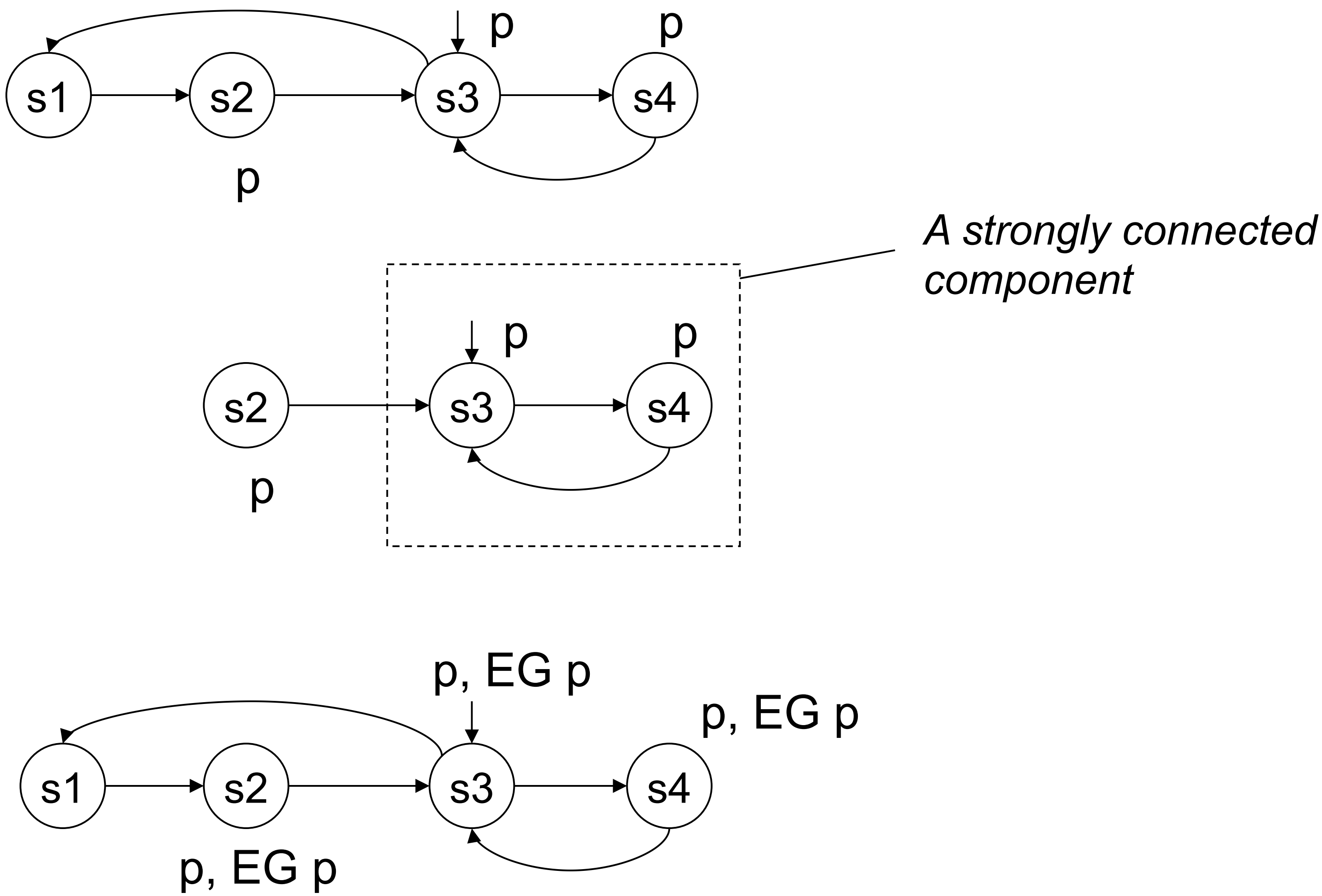




CTL Model Checking Algorithm: EG p

- EG p: Find infinite paths where each node on the path is labeled with p, and label nodes in such paths with EG p
  - First remove all the states which do not satisfy p from the transition graph
  - Compute the strongly connected components of the remaining graph, and then find the nodes which can reach the strongly connected components (both of which can be done in O(|S|+|R|)
  - Label the nodes in the strongly connected components and the nodes that can reach the strongly connected components with EG p

CTL Model Checking Algorithm: EG p



Verification vs. Falsification

- Verification:
  – Show: initial states $\subseteq$ truth set of p

- Falsification:
  – Find: a state $\in$ initial states $\cap$ truth set of $\neg p$
  – Generate a counter-example starting from that state

- Model checking algorithms can be modified to generate a counter-example paths if the property is not satisfied
  – without increasing the complexity

- The ability to find counter-examples is one of the biggest strengths of the model checkers

Counter-Example Generation

- Remember: Given a transition system T= (S, I, R) and a CTL property p T |= p iff for all initial state s $\in$ I, s |= p

- Verification vs. Falsification
  - Verification:
    - Show: initial states $\subseteq$ truth set of *p*
  - Falsification:
    - Find: a state $\in$ initial states $\cap$ truth set of $\neg p$
    - Generate a counter-example starting from that state

- The ability to find counter-examples is one of the biggest strengths of the model checkers

General Idea

- We can define two temporal logics using subsets of CTL operators
  - ACTL: CTL formulas which only use the temporal operators AX, AG, AF and AU and all the negations appear only in atomic properties (there are no negations outside of temporal operators)
  - ECTL: CTL formulas which only use the temporal operators EX, EG, EF and EU and all the negations appear only in atomic properties

- Given an ACTL property its negation is an ECTL property

An Example

- If we wish to check the property AG(p)

- We can use the equivalence:
  $AG(p) \equiv \neg\, EF(\neg p)$

If we can find an initial state which satisfies $EF(\neg p)$, then we know that the transition system T, does not satisfy the property AG(p)

Another Example

- If we wish to check the property AF(p)

- We can use the equivalence:
  $AF(p) \equiv \neg\, EG(\neg p)$

If we can find an initial state which satisfies $EG(\neg p)$, then we know that the transition system T, does not satisfy the property AF(p)

Counter-Example Generation for ACTL

- Given an ACTL property p, we negate it and compute the set of states which satisfy it is negation $\neg$ p
  - $\neg$p is an ECTL property

- If we can find an initial state which satisfies $\neg$ p then we generate a counter-example path for p starting from that initial state by following the states that are marked with $\neg$ p
  - Such a path is called a *witness* for the ECTL property $\neg$ p

Counter-example generation for ACTL

- In general the counter-example for an ACTL property (equivalently a witness to an ECTL property) is not a single path
- For example, the counter example for the property AF(AGp) would be a witness for the property EG(EF¬p)
  - It is not possible to characterize the witness for EG(EF¬p) as a single path
- However it is possible to generate tree-like transition graphs containing counter-example behaviors as a counter-example:
  - Edmund M. Clarke, Somesh Jha, Yuan Lu, Helmut Veith: “Tree-Like Counterexamples in Model Checking”. LICS 2002: 19-29

Counter-example generation for LTL

– Recall that, an LTL property f holds for a transition system T, if and only if, for all execution paths of T, f holds

- Then, to generate a counter-example for an LTL property f, we need to show that there exists an execution path for which $\neg$f holds.
  – Given an LTL property f, a counter-example is an execution path for which $\neg$f holds

What About LTL and CTL* Model Checking?

- The complexity of the model checking problem for LTL and CTL* are:
  - $(|S|+|R|) \times 2^{O(|f|)}$
- Typically the size of the formula is much smaller than the size of the transition system
  - So the exponential complexity in the size of the formula is not very significant in practice