Well-Balanced High Order Methods based on Reconstruction of States

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Abstract: This talk focuses on the numerical approximation of systems of the form

\[
\begin{align*}
U_t + F(U)_x &= B(U)U_x + S(U)H_x, \quad x \in \mathbb{R}, \quad t > 0, \\
U(x, 0) &= U_0(x), \quad x \in \mathbb{R},
\end{align*}
\]

where the unknown \(U(x, t)\) takes values in an open and convex of \(\mathbb{R}^N\); \(F\) and \(S\) are functions from \(\Omega\) to \(\mathbb{R}^N\); \(B\) is a smooth and locally bounded matrix-valued function; and \(H\), a known function from \(\mathbb{R}\) to \(\mathbb{R}\). A number of simplified models for multilayer shallow water or multi-phase fluids can be written in this form. The goal is to present the general framework described in the references below for designing high-order well-balanced shock-capturing numerical methods. The emphasis will be put on the well-balanced property: the numerical schemes are required to exactly solve the smooth stationary solutions of the system. The idea is to extend to high order a first-order path-conservative method (see [5]) by using a reconstruction operator. The main difficulty comes from the fact that, in order to have a well-balanced numerical scheme, this operator has also to preserve the smooth stationary solutions of the system. The strategies presented in [2] and [3] to overcome this difficulty will be presented and some examples will be shown.

REFERENCES


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